Assignment 2

This homework is due Friday Feb 6.

There are total 50 points in this assignment. 45 points is considered 100%. If you go over 45 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 1.3–1.6 of Textbook.

(1) [10pt] Evaluate the following.

(a) $\left|\frac{4-3i}{2-i}\right|$.
(b) $|(1+i)^{2015}|$.
(c) $|z-2|^2$, where $z = x+iy$.
(d) $\mathrm{Arg}(1-i)$.
(e) $\mathrm{Arg}\left(\frac{2}{1-i\sqrt{3}}\right)$.
(f) $\mathrm{Arg}((1+i\sqrt{3})(1+i))$.
(g) $\mathrm{Arg}(\cos 2 - i\sin 2)$.

(2) [10pt] Represent the following complex numbers in polar form (either with sin and cos, or with $e^{i\theta}$, whichever you like more). (*Hint:* Find absolute value first, factor it out, then find argument)

(a) -2015.
(b) $\cos\theta + \sin\theta$.
(c) $6i - 6$.
(d) $-2\sqrt{3} - 2i$.
(e) $\sin\theta - i\cos\theta$.
(f) $-e^{i\theta}$.
(g) $12 - 5i$ (express argument in terms of inverse trig functions).
(h) $-7 - 4i$ (express argument in terms of inverse trig functions).

(3) [5pt] Compute using polar form.
(a) $(i-\sqrt{3})^3(1+i\sqrt{3})^2$.
(b) $\frac{(1+i)^{2015}}{(1-i)^{2013}}$.
(c) $(\sin 3 - i\cos 3)^{-2}(-\cos 1 - i\sin 1)$.

(4) [5pt] Find all roots
(a) of degree 4 of -1.
(b) of degree 6 of $8i$.
(c) of degree 7 of e^{11i}.

(5) [5pt]
(a) Let $z \neq 1$. Show that $1 + z + z^2 + \ldots + z^n = \frac{1-z^{n+1}}{1-z}$.
(b) Compare Re of LHS and RHS in the above equality to show that

$$1 + \cos\theta + \cos 2\theta + \ldots + \cos n\theta = \frac{1}{2} + \frac{\sin\left((n+\frac{1}{2})\theta\right)}{2\sin\frac{\theta}{2}} \text{ for } 0 < \theta < 2\pi.$$

(6) [15pt] Sketch the sets below. In each case state, with reasons, which of the following terms apply to these sets: open; connected; domain; region; closed set; bounded.
(a) $\{z : \mathrm{Re}(z) > 1\}$.
(b) $\{z : -2 \leq \mathrm{Im}(z) < 3\}$.
(c) $\{z : |z - 1 + i| \leq 2\}$.
(d) $\{z : |z + 3i| > 1\}$.
(e) $\{re^{i\theta} : 0 < r < 1 \text{ and } -\frac{\pi}{2} < \theta < \frac{\pi}{2}\}$.
(f) $\{re^{i\theta} : r > 1 \text{ and } \frac{\pi}{4} < \theta < \frac{\pi}{3}\}$.
(g) $\{z : |z| < 1 \text{ or } |z-3| < 1\}$.